

# Основные формулы тригонометрии

$$1. \operatorname{tg} \alpha = \frac{\sin \alpha}{\cos \alpha}$$

$$2. \operatorname{ctg} \alpha = \frac{\cos \alpha}{\sin \alpha}$$

$$3. \cos^2 \alpha + \sin^2 \alpha = 1$$

$$4. \operatorname{tg} \alpha = \frac{1}{\operatorname{ctg} \alpha}$$

$$5. \operatorname{ctg} \alpha = \frac{1}{\operatorname{tg} \alpha}$$

$$6. \operatorname{tg} \alpha \cdot \operatorname{ctg} \alpha = 1$$

$$7. 1 + \operatorname{ctg}^2 \alpha = \frac{1}{\sin^2 \alpha}$$

$$8. 1 + \operatorname{tg}^2 \alpha = \frac{1}{\cos^2 \alpha}$$

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$$9, 10. \sin(\alpha \pm \beta) = \sin \alpha \cdot \cos \beta \mp \cos \alpha \cdot \sin \beta$$

$$11, 12. \cos(\alpha \pm \beta) = \cos \alpha \cdot \cos \beta \mp \sin \alpha \cdot \sin \beta$$

$$13, 14. \operatorname{ctg}(\alpha \pm \beta) = \frac{\operatorname{ctg} \alpha \cdot \operatorname{ctg} \beta \mp 1}{\operatorname{ctg} \beta \pm \operatorname{ctg} \alpha}$$

$$15, 16. \operatorname{tg}(\alpha \pm \beta) = \frac{\operatorname{tg} \alpha \pm \operatorname{tg} \beta}{1 \mp \operatorname{tg} \alpha \cdot \operatorname{tg} \beta}$$

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$$17. \sin 2\alpha = 2 \sin \alpha \cdot \cos \alpha$$

$$18. \cos 2\alpha = 1 - 2 \sin^2 \alpha$$

$$19. \cos 2\alpha = 2 \cos^2 \alpha - 1$$

$$20. \cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$$

$$21. \sin 3\alpha = 3 \sin \alpha - 4 \sin^3 \alpha$$

22.

$$\sin 4\alpha = 8 \cos^3 \alpha \cdot \sin \alpha - 4 \cos \alpha \cdot \sin \alpha$$

$$23. \cos 3\alpha = 4 \cos^3 \alpha - 3 \cos \alpha$$

$$24. \cos 4\alpha = 8 \cos^4 \alpha - 8 \cos^2 \alpha + 1$$

$$25. \operatorname{tg} 2\alpha = \frac{2 \operatorname{tg} \alpha}{1 - \operatorname{tg}^2 \alpha}$$

$$26. \operatorname{ctg} 2\alpha = \frac{\operatorname{ctg} \alpha - 1}{2 \operatorname{ctg} \alpha}$$

$$27. \operatorname{tg} 3\alpha = \frac{3 \operatorname{tg} \alpha - \operatorname{tg}^3 \alpha}{1 - 3 \operatorname{tg}^2 \alpha}$$

$$28. \operatorname{ctg} 3\alpha = \frac{\operatorname{ctg}^3 \alpha - 3 \operatorname{ctg} \alpha}{3 \operatorname{ctg}^2 \alpha - 1}$$

$$29. \operatorname{tg} 4\alpha = \frac{4 \operatorname{tg} \alpha - 4 \operatorname{tg}^3 \alpha}{1 - 6 \operatorname{tg}^2 \alpha + \operatorname{tg}^4 \alpha}$$

$$30. \operatorname{ctg} 4\alpha = \frac{\operatorname{ctg}^4 \alpha - 6 \operatorname{ctg}^2 \alpha + 1}{4 \operatorname{ctg}^3 \alpha - 4 \operatorname{ctg} \alpha}$$

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$$31. \sin \frac{\alpha}{2} = \sqrt{\frac{1 - \cos \alpha}{2}}$$

$$32. \cos \frac{\alpha}{2} = \sqrt{\frac{1 + \cos \alpha}{2}}$$

$$33, 34. \operatorname{tg} \frac{\alpha}{2} = \frac{\sin \alpha}{1 + \cos \alpha} = \frac{1 - \cos \alpha}{\sin \alpha}$$

$$35, 36. \operatorname{ctg} \frac{\alpha}{2} = \frac{\sin \alpha}{1 - \cos \alpha} = \frac{1 + \cos \alpha}{\sin \alpha}$$

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$$37. \sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cdot \cos \frac{\alpha - \beta}{2}$$

$$38. \sin \alpha - \sin \beta = 2 \sin \frac{\alpha - \beta}{2} \cdot \cos \frac{\alpha + \beta}{2}$$

$$39. \cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cdot \cos \frac{\alpha - \beta}{2}$$

$$40. \cos \alpha - \cos \beta = -2 \sin \frac{\alpha + \beta}{2} \cdot \sin \frac{\alpha - \beta}{2}$$

$$41. \operatorname{tg} \alpha + \operatorname{tg} \beta = \frac{\sin(\alpha + \beta)}{\cos \alpha \cdot \cos \beta}$$

$$42. \operatorname{tg} \alpha - \operatorname{tg} \beta = \frac{\sin(\alpha - \beta)}{\cos \alpha \cdot \cos \beta}$$

$$43. \operatorname{ctg} \alpha + \operatorname{ctg} \beta = \frac{\sin(\alpha + \beta)}{\sin \alpha \cdot \sin \beta}$$

$$44. \operatorname{ctg} \alpha - \operatorname{ctg} \beta = \frac{-\sin(\alpha - \beta)}{\sin \alpha \cdot \sin \beta}$$

$$45. \cos \alpha + \sin \alpha = \sqrt{2} \cdot \cos(45^\circ - \alpha)$$

$$46. \cos \alpha - \sin \alpha = \sqrt{2} \cdot \sin(45^\circ - \alpha)$$

$$47. \operatorname{tg} \alpha + \operatorname{ctg} \beta = \frac{\cos(\alpha - \beta)}{\cos \alpha \cdot \sin \beta}$$

$$48. \operatorname{tg} \alpha - \operatorname{ctg} \beta = \frac{-\cos(\alpha + \beta)}{\cos \alpha \cdot \sin \beta}$$

$$49. \operatorname{tg} \alpha - \operatorname{ctg} \alpha = -2 \operatorname{tg} 2\alpha$$

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$$50. 1 + \cos \alpha = 2 \cos^2 \frac{\alpha}{2}$$

$$51. 1 - \cos \alpha = 2 \sin^2 \frac{\alpha}{2}$$

$$52. 1 + \sin \alpha = 2 \cos^2(45^\circ - \frac{\alpha}{2})$$

$$53. 1 - \sin \alpha = 2 \sin^2(45^\circ - \frac{\alpha}{2})$$

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$$54. \sin^2 \alpha = \frac{1 - \cos 2\alpha}{2}$$

$$55. \sin^3 \alpha = \frac{1}{4}(3\sin \alpha - \sin 3\alpha)$$

$$56. \cos^2 \alpha = \frac{1 + \cos 2\alpha}{2}$$

$$57. \cos^3 \alpha = \frac{1}{4}(\cos 3\alpha + 3\cos \alpha)$$

$$58. \sin^4 \alpha = \frac{1}{8}(\cos 4\alpha - 4\cos 2\alpha + 3)$$

$$59. \cos^4 \alpha = \frac{1}{8}(\cos 4\alpha + 4\cos 2\alpha + 3)$$

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$$60. \sin \alpha \cdot \cos \beta = \frac{1}{2}(\sin(\alpha + \beta) + \sin(\alpha - \beta))$$

$$61. \cos \alpha \cdot \cos \beta = \frac{1}{2}(\cos(\alpha + \beta) + \cos(\alpha - \beta))$$

$$62. \sin \alpha \cdot \sin \beta = \frac{1}{2}(\cos(\alpha - \beta) - \cos(\alpha + \beta))$$

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$$63. \arcsin x = -\arcsin(-x) = \frac{\pi}{2} - \arccos x = \operatorname{arctg} \frac{x}{\sqrt{1-x^2}}$$

64.

$$\arccos x = \pi - \arccos(-x) = \frac{\pi}{2} - \arcsin x = \operatorname{arctg} \frac{x}{\sqrt{1-x^2}}$$

65.

$$\operatorname{arctg} x = -\operatorname{arctg}(-x) = \frac{\pi}{2} - \operatorname{arccot} x = \arcsin \frac{x}{\sqrt{1+x^2}}$$

66.

$$\operatorname{arccot} x = \pi - \operatorname{arccot}(-x) = \frac{\pi}{2} - \operatorname{arctg} x = \arccos \frac{x}{\sqrt{1+x^2}}$$

$$67. \arcsin(-x) = -\arcsin x$$

$$68. \arccos(-x) = \pi - \arccos x$$

$$69. \operatorname{arctg}(-x) = -\operatorname{arctg} x$$

$$70. \operatorname{arccot}(-x) = \pi - \operatorname{arccot} x$$

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$$71. \cos \alpha = \frac{1 - \operatorname{tg}^2 \frac{\alpha}{2}}{1 + \operatorname{tg}^2 \frac{\alpha}{2}}$$

$$72. \sin \alpha = \frac{2 \operatorname{tg} \frac{\alpha}{2}}{1 + \operatorname{tg}^2 \frac{\alpha}{2}}$$

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$$73. \begin{aligned} \sin x = a; |a| < 1 \\ x = (-1)^n \arcsin a + \pi n, n \in \mathbb{Z} \end{aligned}$$

$$74. \begin{aligned} \cos x = a; |a| < 1 \\ x = \pm \arccos a + 2\pi n, n \in \mathbb{Z} \end{aligned}$$

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$$75. \begin{aligned} & \operatorname{tg} x = a \\ & x = \operatorname{arctg} a + \pi n, n \in \mathbb{Z} \end{aligned}$$

$$76. \begin{aligned} & \operatorname{ctg} x = a \\ & x = \operatorname{arcctg} a + \pi n, n \in \mathbb{Z} \end{aligned}$$

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$$77. \begin{aligned} & \text{при } |a| \leq 1 \\ & 0 \leq \arccos a \leq \pi \\ & \cos(\arccos a) = a \end{aligned}$$

$$78. \begin{aligned} & \text{при } |a| \leq 1 \\ & -\frac{\pi}{2} \leq \arcsin a \leq \frac{\pi}{2} \\ & \sin(\arcsin a) = a \end{aligned}$$

$$79. \begin{aligned} & \text{при } \forall a \\ & -\frac{\pi}{2} < \operatorname{arctg} a < \frac{\pi}{2} \\ & \operatorname{tg}(\operatorname{arctg} a) = a \end{aligned}$$

$$80. \begin{aligned} & \text{при } \forall a \\ & 0 < \operatorname{arcctg} a < \pi \\ & \operatorname{ctg}(\operatorname{arcctg} a) = a \end{aligned}$$

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$$81. \arcsin a + \arccos a = \frac{\pi}{2}$$

$$82. \operatorname{arctg} a + \operatorname{arcctg} a = \frac{\pi}{2}$$

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$$83. \begin{aligned} & \sin x = 0 \\ & x = \pi n, n \in \mathbb{Z} \end{aligned}$$

$$84. \begin{aligned} & \cos x = 0 \\ & x = \frac{\pi}{2} + \pi n, n \in \mathbb{Z} \end{aligned}$$

$$85. \begin{aligned} & \sin x = 1 \\ & x = \frac{\pi}{2} + 2\pi n, n \in \mathbb{Z} \end{aligned}$$

$$86. \begin{aligned} & \cos x = 1 \\ & x = 2\pi n, n \in \mathbb{Z} \end{aligned}$$

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$$87. \begin{aligned} & \arcsin x = a \\ & -\frac{\pi}{2} \leq a \leq \frac{\pi}{2} \\ & x = \sin a \end{aligned}$$

$$88. \begin{aligned} & \arccos x = a \\ & 0 \leq a \leq \pi \\ & x = \cos a \end{aligned}$$

$$\operatorname{arctg} x = a$$

$$89. -\frac{\pi}{2} < a < \frac{\pi}{2}$$

$$x = \operatorname{tga}$$

$$\operatorname{arcctg} x = a$$

$$90. 0 \leq a \leq \pi$$

$$x = \operatorname{ctga}$$